Digital Image Processing

Image Restoration:

Noise Removal

Image restoration

- Image restoration is the process of recovering the original scene from the observed scene which is degraded.

- Different form enhancement- aim of enhancement techniques make images visually appealing, whereas restoration essentially inverts the degradation- more objective.

Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging sensors can be affected by ambient conditions
- Interference can be added to an image during transmission

Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

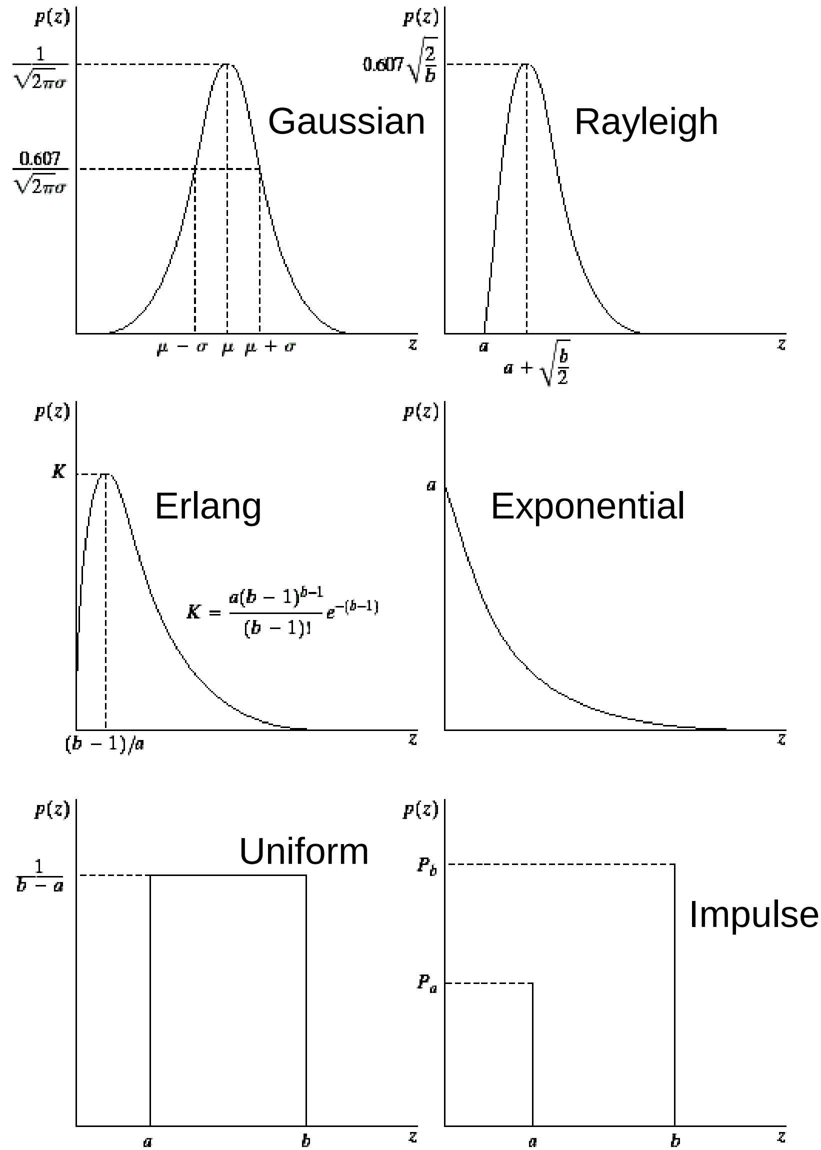
- If we can estimate the model of the noise in an image, this will help us to figure out how to restore the image

- Noise is assumed to be uncorrelated with pixel values and does not depend on spatial coordinates

Noise Models

There are many different models for the image noise term $\eta(x, y)$:

- Gaussian
  - Most common model
- Rayleigh
- Erlang
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise



Noise models

- Noise is characterized by its probability density function. Some of which are:
- Gaussian, Rayleigh, Uniform, Exponential, etc.
- By far Gaussian is the most popular model, because:
  - It is present widely in practice
  - Mathematical ease

Filtering to Remove Noise

We can use spatial filters of different kinds to remove different kinds of noise

The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

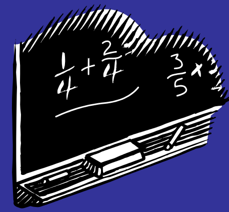
This is implemented as the simple smoothing filter

Blurs the image to remove noise

Other Means

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean
- Harmonic Mean
- Contraharmonic Mean



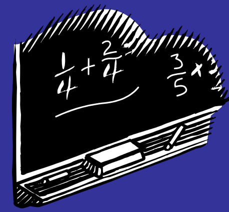
Other Means (cont…)

There are other variants on the mean which can give different performance

Geometric Mean:

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail



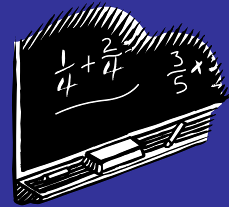
Other Means (cont…)

Harmonic Mean:

$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise

Also does well for other kinds of noise such as Gaussian noise



Other Means (cont…)

Contraharmonic Mean:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter and adjusting its value changes the filter's behaviour

Positive values of Q eliminate pepper noise

Negative values of Q eliminate salt noise

Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter

Useful spatial filters include

- Median filter
- Max and min filter
- Midpoint filter
- Alpha trimmed mean filter

Median Filter

Median Filter:

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters

Particularly good when salt and pepper noise is present

Max and Min Filter

Max Filter:

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter:

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max filter is good for pepper noise and min is good for salt noise

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random Gaussian and uniform noise

Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

We can delete the $d/2$ lowest and $d/2$ highest grey levels

So $g_r(s, t)$ represents the remaining $mn - d$ pixels

Adaptive Filters

- The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another
- The behaviour of **adaptive filters** changes depending on the characteristics of the image inside the filter region
- We will take a look at the **adaptive median filter**

Adaptive filters for handling noise

- Adapts to some statistical measures on a local neighborhood, usually mean and variance:
  - g(i,j): the noisy image gray-level value.
  - σ_n^2 : noise variance in the image.
  - m_L : local mean in neighborhood.
  - σ_L^2 : local variance in neighborhood.

- σ_n^2 could be estimated from an uniform area in the given image.

Adaptive local noise reduction filter

- If noise variance is zero → Indicates no noise → return the observed image.
- If local variance is high compared to the noise variance → presence of edge or a sharp feature→ return a value close to observed grey-value.
- If two variances are equal→ presence of noise→ return the arithmetic mean

Adaptive local noise reduction filter

- Filter equation:
  - $$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$
- How to handle $\sigma_\eta^2 > \sigma_L^2$, what does it mean?
- When $\sigma_\eta^2 > \sigma_L^2$, let $\frac{\sigma_\eta^2}{\sigma_L^2} = 1$ in the filter equation.

Adaptive Median Filtering

- The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large

- The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise

- The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image

Adaptive Median Filtering (cont…)

- Remember that filtering looks at each original pixel image in turn and generates a new filtered pixel
- First examine the following notation:
  - z_{min} = minimum grey level in S_{xy}
  - z_{max} = maximum grey level in S_{xy}
  - z_{med} = median of grey levels in S_{xy}
  - z_{xy} = grey level at coordinates (x, y)
  - S_{max} =maximum allowed size of S_{xy}

Adaptive Median Filtering (cont…)

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1 > 0$ and $A2 < 0$, Go to level B

Else increase the window size

If window size $\leq S_{max}$ repeat level A

Else output z_{xy}

Level B: $B1 = z_{xy} - z_{min}$

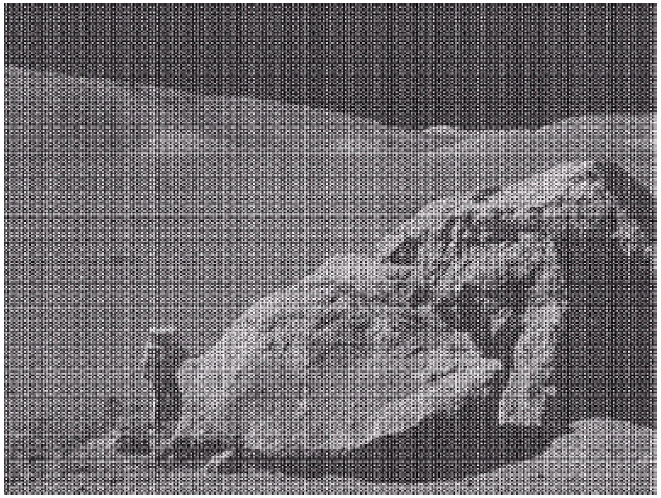
$B2 = z_{xy} - z_{max}$

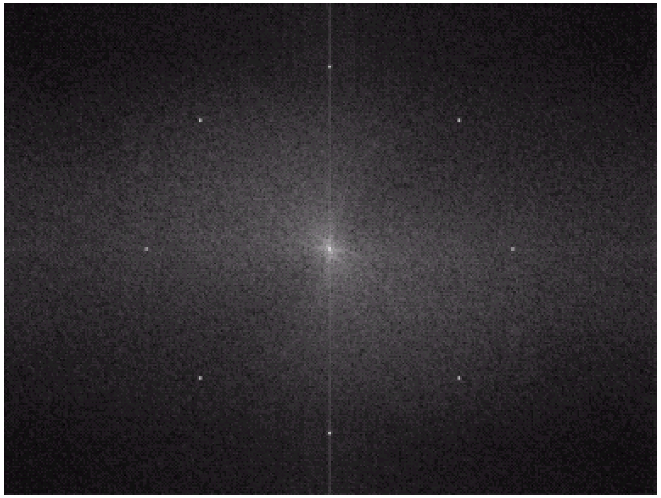
If $B1 > 0$ and $B2 < 0$, output z_{xy}

Else output z_{med}

Periodic Noise

- It arises due to electrical or electromagnetic interference
- Gives rise to regular noise patterns in an image
- Frequency domain techniques in the Fourier domain are most effective at removing periodic noise





Band Reject Filters

- Removing periodic noise form an image involves removing a particular range of frequencies from that image
- *The reject* filters can be used for this purpose

An ideal band reject filter is given as follows:

$$H(u,v) = \begin{cases} 1 & if\ D(u,v) < D_0 - \dfrac{W}{2} \\ 0 & if\ D_0 - \dfrac{W}{2} \le D(u,v) \le D_0 + \dfrac{W}{2} \\ 1 & if\ D(u,v) > D_0 + \dfrac{W}{2} \end{cases}$$